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MODEL-BUILDING OF CROP-ROTATION DATA WITH AUTOREGRESSIVE ERROR STRUCTURE

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SUMMARY

Multiple regression analysis of four crop-rotation experiments on rice indicated insignificance of a few individual regression coefficients, although the models were having a high and significant predictability. The direct analysis of residuals indicated autocorrelation of residues and hence regression models are found inappropriate for making any further analysis. Based on Durbin-Watson's [2] test procedure the serial correlations were tested and found significant and hence the errors have been distributed with a first-order autoregressive error structure. An error model which belongs to the class of autoregressive moving average (ARMA) models as discussed by Box and Jenkins [1] has been proposed and calibrated for the data of plot-wise residues and examined for its relative efficiency over a first-order autoregressive error structure for refining both prediction of rice yields and optimisation of soil and fertiliser nutrients in a black soil.

Key words : Crop-rotation, Autocorrelation, Autoregressive structure, Relative efficiency, Error model.

Introduction

One of the crucial assumptions made in the process of model-building is that the errors are serially independent and are distributed normal with mean zero and variance $\hat{\sigma}^2$. When errors are autocorrelated with each other, the regression estimates are inefficient and biased downwards,

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if autocorrelation is positive and biased upwards, if autocorrelation is negative. In the presence of autocorrelated errors, the regression models are also inappropriate for making either yield prediction or nutrient optimisation. Even if the coefficient of prediction (R^2) is high and significant, the estimates of regression coefficients based on a model would become low and insignificant and with misleadingly small variance, if the errors are autocorrelated with each other. The autocorrelations can be tested based on Durbin-Watson's [2] test procedure.

We can consider an error model with a first-order autoregressive error component of plot-wise residues for predicting post harvest soil test values of a nutrient in the standard Soil Test Crop Response (*STCR*) model in which the post harvest soil test value (*STV* (*PH*)) of a plot is a function of initial soil test value (*STV* (*I*)), grain yield (*Y*) and fertiliser nutrient (*F*) and can be given as

$$STV\,(PH) = f\,(Y,\ STV\,(I),\ F) \tag{1}$$

In a crop-rotation system, where a minimum of at least two crops are involved in two different seasons, the first-order autoregressive moving average (ARMA) models are most appropriate for plot-wise errors. Using model (1), the prediction of soil test values is made for each crop-rotation.

Patterson and Lowe [3] have introduced an error model which is a particular case of Box and Jenkin's [1] ARMA models for accounting diminishing autocorrelations. An analysis of residues has been made in this paper using ARMA models for data of crop-rotation experiments for distributing error structure. The error models derived for two rotations are for kharif rice followed by rabi rice in kharif-rabi rotation, and rabi rice followed by kharif rice in rabi-kharif rotation respectively. An error model has been proposed for representing plotwise residues of soil *N*, *P* and *K* nutrients. The model has been compared with a first-order autoregressive scheme for drawing inferences about relative efficiency of a model under each crop-rotation.

Materials and Methods

Estimation of Autocorrelation

Using a first-order auto-regressive relationship of residues, viz.,

$$u_t = \rho u_{t-1} + v_t \tag{2}$$

an estimate 'r' of population autocorrelation 'ρ' can be given as

$$r = \frac{\sum_{2}^{n} u_t u_{t-1}}{\sum_{2}^{n} u_t^2 \sum_{2}^{n} u_{t-1}^2} \tag{3}$$

where u's are residues and v's are distributed normal with mean 0 and variance σ_v^2.

The prediction model for soil N can be given as

$$SN\,(PH) = A + BY + C\,SN\,(I) + D\,FN \tag{4}$$

where $SN\,(PH)$, $SN\,(I)$, Y and FN are as given in (1) and A, B, C, D are regression coefficients.

The model (4) is calibrated separately for soil N, P and K nutrients under each rotation and residues based on each model are analysed for refining models with different error structures.

Testing of Autocorrelation

The hypothesis that residues are not autocorrelated with a first-order autoregressive scheme can be tested by using Watson's test statistic as

$$d^* = \frac{\sum_{2}^{n} (e_t - e_{t-1})^2}{\sum_{1}^{n} e_t^2} \tag{5}$$

We have to compare the d^* value with the value of upper limit (d_u) and the value of lower limit (d_L) as given in Durbin-Watson tables for testing the significance of d^* values and draw conclusions accordingly.

Autoregressive Model for Residues

For distributing residues to first-order autoregressive scheme, as given in (2), the appropriate transformation is to subtract from original observations, the product of $\hat{\rho}$ times the value of previous observations. We apply ordinary least squares to the transformed model

$$SN\,(PH)_t^* = A_1 + B_1\,Y_t^* + C_1\,SN\,(I)_t^* + D_1\,FN_t^* \tag{6}$$

where $SN(PH)_t^* = SN(PH)_t - \hat{\rho}\, SN(PH)_{(t-1)}$

$$Y_t^* = Y_t + \hat{\rho}\, Y_{(t-1)}$$

$$SN(I)_t^* = SN(I)_t - \hat{\rho}\, SN(I)_{(t-1)}$$

$$FN_t^* = FN_t - \hat{\rho}\, FN_{(t-1)}.$$

Given a pair of soil fertility blocks, estimates of error (θ) based on model (6) for soil nutrients can be given as

$$\theta = MW \qquad (7)$$

where

$$\theta = \begin{pmatrix} \theta 1 \\ \theta 2 \\ \theta 3 \end{pmatrix}_{3\times 1} ; \quad M = \begin{bmatrix} M1 & M_1^* \\ M2 & M_2^* \\ M3 & M_3^* \end{bmatrix}_{3\times 2} \quad \text{and } W = \begin{pmatrix} W1 \\ W2 \end{pmatrix}_{2\times 1}$$

Here M represents mean residues of different blocks and W represents weights of each residual component, viz., $W = 1 - C$ where C is ratio of number of outlying plots to total number of plots.

A more general model for errors of soil fertility of nutrients (ψ) can be given as

$$\psi = S\phi + Z\xi \qquad (8)$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}_{3\times 1} ; \quad S = \begin{bmatrix} S_1 & S_1^* \\ S_2 & S_2^* \\ S_3 & S_3^* \end{bmatrix}_{3\times 2} ; \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}_{2\times 1} ;$$

$$Z = \begin{bmatrix} Z_1 & Z_1^* \\ Z_2 & Z_2^* \\ Z_3 & Z_3^* \end{bmatrix}_{3\times 2} \quad \text{and} \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}_{2\times 1}$$

Here S and Z represent mean residues of each block under kharif-rabi and rabi-kharif rotations for each nutrient and ϕ and ξ are relative weights of each residual component as discussed earlier.

The residues based on models (7) and (8) are compared for each nutrient under each rotation and conclusions are drawn with regard to relative efficiency of a model. The percentage relative efficiency of a model A to model B can be given as

$$\text{PRE}(A) = \frac{V_B}{V_A} \times 100 \tag{9}$$

where V_A and V_B are combined error variances of model A and model B based on relative weights as given earlier. An error model A will be preferred to error model B, if PRE (A) is greater than 100, rejected if PRE (A) is less than 100. If PRE (A) is equal to 100, then experimenter may choose either of the two equally efficient error models.

Results and Discussion

Experimental Data

Four field experiments on Rice (*Oryza sativa*) were conducted in a black soil of Agricultural Research Institute, Rajendranagar, Hyderabad during kharif and rabi seasons of 1979 and 1980 under two different rotations for studying the efficient use of integrated nutrient supply for rice-rice rotation system (Prasada Rao, 1984). The rotations taken for study are kharif rice followed by rabi rice (kharif-rabi rotation) and rabi rice followed by kharif rice (rabi-kharif rotation). There were 6 blocks in each experiment, 3 blocks representing Farm Yard Manure (FYM) series (F 1) and 3 blocks representing no Farm Yard Manure series (F 0). In $F1$ blocks, FYM was uniformly applied in order to supply N at the rate of 20 kg/ha. Each block was divided into 24 plots for superimposing fertiliser treatments of N, P and K nutrients generated based on an assymetrical factorial design of 4, 3 and 2 levels respectively.

Soil samples were collected from each plot before superimposing fertiliser treatments and analysed for soil N, P and K nutrients. At harvest, observations on grain yield were taken in each plot. There was a wide range in estimates of soil nutrients and grain yield in each experiment.

An Autoregressive Model of Soil Fertility

Regression models as given in (4) were calibrated for each nutrient for predicting post-harvest soil test values separately under $F0$ and $F1$ series for each rotation. The estimates of predictability (R^2) were found low and non-significant for all nutrients under both rotations. The residues were analysed and estimates of autocorrelation were determined for each nutrient. Based on test procedure as given in (5), the autocorrelations were found highly significant for all nutrients under both rotations.

A set of 13 outlying plots for kharif-rabi rotation and 14 outlying plots for rabi-kharif rotation which were common for all nutrients have been identified and eliminated for refining soil fertility predictions. The revised regressions were found to significantly increase R^2 and also decrease experimental error ($\hat{\sigma}$) under a given model. There was a significant decrease in estimates of autocorrelation of soil fertility residues of all nutrients under revised models. The estimates of revised regression coefficients and R^2 and $\hat{\sigma}$ values along with autocorrelations of both original and revised regression models are given in Table 1. Based on the analysis, the first-order autoregressive scheme was found adequate for making significant error predictions since the model has significantly reduced estimates of autocorrelation. Using model (7), combined estimates of error of each soil nutrient, θ, are derived under each rotation and are given in Table 2.

A Proposed Error Model

The matrix S of estimates of mean residues for kharif predictions in terms of rabi parameters, and matrix Z of mean residues for rabi predictions in terms of kharif parameters have been derived. Using relative weights ϕ and ξ for each error component, an estimate of error of each soil nutrient i.e., ψ has been worked out for each rotation based on error model as given in (8). The estimates ψ were found low when compared with estimates θ as derived using model (7). The combined estimates of error of soil fertility of each nutrient along with relative weights under each rotation are given in Table 2.

In the above discussion, the errors θ and ψ are comparable due to the fact that they are distributed normal with mean 0 and variance σ^2. Further, the error model (8) was found to provide lower estimates of error due to better distribution of mean residues under kharif-rabi and rabi-kharif rotations and also due to completeness of the model.

Comparison of Different Error Models

The error models proposed in (7) and (8) have been compared for all nutrients by using percentage relative efficiency criteria as given in (9). The combined error variances have been derived by using relative weights of a nutrient in a given model under a given rotation. Based on the above analysis, the error model as proposed in (8) was found to provide lower estimates of error for all the three nutrients and hence was relatively more efficient in both kharif-rabi and rabi-kharif rotations when compared with error model as postulated in (7). The percentage relative

TABLE 1—ESTIMATES OF AUTOCORRELATION, SOIL FERTILITY REGRESSION COEFFICIENTS AND EXPERIMENTAL ERROR

Category	Nutrient	*Original estimates*				*Revised soil fertility regression coefficients*				*Revised estimates*			
		AC	*DW*	R^2	$\hat{\sigma}$	*Intercept*	*Yield*	*Soil nutrient*	*Fertiliser nutrient*	*AC*	*DW*	R^2	$\hat{\sigma}$
K–R	*N*	0.26	5.56	0.22	13.78	383**	−0.0077	−0.0607*	0.0009	0.10	4.21	0.50**	8.75
F0	*P*	−0.27	6.30	0.43	11.10	46**	−0.0029**	0.4254**	0.0410*	−0.12	8.61	0.78**	3.47
	K	−0.11	9.77	0.25	24.74	347 *	−0.0144**	0.1698 *	0.0077	−0.02	5.96	0.59**	13.07
F1	*N*	0.29	7.91	0.28	24.51	279**	0.0334**	−0.1447**	−0.3293**	0.09	5.19	0.61**	13.15
	P	0.07	9.92	0.37	5.04	63**	0.0011*	−0.1914**	0.0798**	0.02	3.12	0.73**	2.55
	K	0.14	7.58	0.40	20.99	262**	−0.0023	0.2673**	0.2958**	0.11	5.49	0.71**	11.28
R–K	*N*	−0.07	8.84	0.08	14.38	219**	−0.0048*	−0.0463**	0.0909	−0.01	5.75	0.54**	8.80
F0	*P*	0.04	10.90	0.08	1.58	7**	0.0001	0.1464**	0.0068*	0.01	4.69	0.62**	0.77
	K	0.38	4.75	0.14	12.74	434**	0.0004	0.0244**	0.1604**	0.08	5.51	0.75**	7.59
F1	*N*	0.28	6.34	0.01	16.47	218**	−0.0026	−0.0678*	0.0065	0.11	6.25	0.58**	9.12
	P	0.25	6.59	0.21	2.71	6**	0.0011	0.5769**	−0.0083	0.09	4.38	0.71**	1.52
	K	0.15	8.60	0.42	14.43	358**	0.0013	0.1943**	0.2922**	0.05	6.95	0.78**	7.58

AC : Autocorrelation; DW : Durbin-Watson test; $\hat{\sigma}$: Experimental error; K—R : kharif-rabi; R—K : Rabi-kharif rotation.

TABLE 2—ESTIMATES OF ERROR COMPONENTS AND PERCENTAGE RELATIVE EFFICIENCIES OF DIFFERENT ERROR MODELS

Model	*Statistic*	*Soil N*	*Soil P*	*Soil K*
First-order Autoregressive Model (K—R)	$W1$	0.8194	0.8194	0.8194
	$W2$	0.8194	0.8194	0.8194
	M	−0.0008	−0.0005	−0.0002
	M^*	−0.0023	−0.0002	0.0006
	$\hat{\sigma}^2$	402.0000	75.0000	535.0000
	PRE	32.0000	12.0000	28.0000
First-order Autoregressive Model (R—K)	$W1$	0.8055	0.8055	0.8055
	$W2$	0.8055	0.8055	0.8055
	M	−0.0002	−0.0001	0.0001
	M^*	−0.0003	−0.0001	0.0024
	$\hat{\sigma}^2$	243.0000	5.0000	188.0000
	PRE	34.0000	40.0000	31.0000
Proposed Model (K—R)	$\phi 1$	0.8194	0.8194	0.8194
	$\phi 2$	0.8194	0.8194	0.8194
	S	−0.0004	−0.0001	−0.0001
	S^*	−0.0009	−0.0001	0.0002
	$\hat{\sigma}^2$	127.0000	9.0000	152.0000
	PRE	316.0000	833.0000	352.0000
Proposed Model (R—K)	$\xi 1$	0.8055	0.8055	0.8055
	$\xi 2$	0.8055	0.8055	0.8055
	Z	−0.0001	−0.0001	0.0001
	Z^*	−0.0001	−0.0001	0.0006
	$\hat{\sigma}^2$	82.0000	2.0000	59.0000
	PRE	296.0000	250 0000	318.0000

efficiencies of soil fertility error predictions of N, P and K nutrients based on models (7) and (8) are given in Table 2. Since percentage relative efficiencies of N, P and K models are greater than 100, the proposed error model is preferred to first-order autoregressive model for all nutrients. The relative efficiencies have also indicated usefulness of revised regressions after eliminating outlying plots and estimating error based on kharif and rabi prediction models.

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